PL-TR-94-2001

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WORLD HEIGHT SYSTEM DEFINTION AND IMPLEMENTATION

Richard H. Rapp
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The Ohio State University
Department of Geodetic Science and Surveying
Columbus, OH 43210

December 1993

Final Report
9 September 1992 - 11 December 1993

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ERRATA

Cover: In the title, the word "Defintion" should read "Definition."

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REPORT DOCUMENTATION PAGE

Form Approved
OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
	December 1993	Final Report 9/9/92 - 12/11/93

4. TITLE AND SUBTITLE

World Height System Definition and Implementation

5. FUNDING NUMBERS

F19628-92-K-0026

PR 3204
TA DM
WU A0

6. AUTHOR(S)

Richard H. Rapp and Nagarajan Balasubramania

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)

Department of Geodetic Science and Surveying
The Ohio State University
Columbus, Ohio 43210

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)

Phillips Laboratory
29 Randolph Road
Hanscom Air Force Base, MA 01731-3010

Contract Manager: Anestis Romaides/GPEG

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

PL-TR-94-2001

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

Approved for public release; distribution unlimited

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

Height systems of the world are inconsistent at the ±1 m level of accuracy. This report discusses two procedures that attempt to determine an ideal equipotential surface for heights and the connection of regional vertical datums. One method uses 18 stations whose precise geocentric coordinates are known along with the orthometric or mean sea level height with respect to a regional vertical datum. This information is combined with gravity data (for geoid undulation determination) in a general least squares adjustment. The results include.the separation between a reference ellipsoid and the ideal (geoid) surface as well as the connection of the regional datums to the geoid. One finds, for example, the origin of the NAVD88 system (U.S.) is 40 ±4 cm below the geoid while the ODN (Newlyn) (England) is 26 ±23 cm below the geoid.

A second procedure to examine the difference between vertical datums used Doppler positioned stations on various vertical datums. Using a degree 360 potential coefficient model, for geoid undulation determination, the separation between several datums was determined. For example, the NN datum used in Germany appears to be 91 cm higher than the reference surface of the ODN (Newlyn) datum used in England.

The methods and results described in this report form a foundation for the development of a world height system.

14. SUBJECT TERMS

Height Systems, Geoid, Vertical Datum, Gravity

15. NUMBER OF PAGES

34

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
unclassified	unclassified	unclassified	SAR

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1. Introduction

This report is the final report under this project that was started on September 11, 1992. The research has been carried out under Grant F19628-92-K-0026 from the Phillips Laboratory at the Hanscom AFB, MA. The funding was awarded in response to a proposal submitted to the Lab on May 21, 1992. The original ending date of the grant was 8 August 1993. It was later extended to 11 December 1993.

There were two main areas of study that were outlined in the proposal. These were:

A. Height Bias Model for Conversion of GPS/Geoid Information to Local Vertical Datum

1. Select two geographic regions in the U.S. and/or Canada for study. Acquire needed data from appropriate organizations.
2. Develop height bias model after examining height inconsistencies assuming all data is correct.
3. Carry out estimation of height bias model for two cases: one using absolute height determinations and another using relative height determinations.
4. Discuss applications of modeling bias function with dependency on data distribution.
5. Define recommended procedures for bias modeling under different data scenarios.

B. World Height System Definition Through Space Positioning

1. Extend global simulation studies to incorporate local areas in which no global network stations are available.
2. Implement a test calculation to provide a vertical datum connection between two well defined vertical datums. We would propose that the two systems be UELN73 and NAVD88.
3. Develop recommendations for the acquisitions and use of data for the determination of a World Height System.

During this project time period, substantial research has been carried out for the two main areas of study noted above. In addition, a study was carried out that used a large station position data set to attempt a vertical datum analysis. The results of these studies will be reported in this final technical report of the project.

2. Research Activities and Findings

The research under this project was carried out by Richard H. Rapp and Nagarajan Balasubramania. Some of this research followed ideas developed in the report by Rapp and Balasubramania (1992). For convenience, this final report has been divided into sections prepared by each author.

2.1 Research Activities of N. Balasubramania

In this project research has been carried out in the two areas described in Section 1. The research in the topics of Section A were previously described in Status Report No. 1 of this project which was prepared in January 1993. For completeness, the material presented in the previous report will also be presented here. The research under Item B has not been previously reported so that the information to be given here is entirely new.

2.1.1 Studies in Height Bias Model Conversion

The initial studies in the project were related to the development of a height bias model using GPS heights, leveling information, and precise geoid undulation information. The research was concentrated mainly on the development of an height bias model that can facilitate the conversion of geoid/GPS information to local vertical datum. Since the regional vertical datums that are realized and used today do not have the ideal geoid as reference surface, the orthometric heights computed on a regional vertical datum will be different from the orthometric height that is computed with respect to an ideal geoid. The difference may simply be a bias or it may be a difference driven by a number of factors including distortion in a local vertical network (Rapp, 1992). Assuming a linear relation between the orthometric height computed in a local vertical datum H_D and the orthometric height H_I computed using the relation,

$$H_I = h - N \tag{1}$$

where h is the ellipsoidal height of the station measured along the ellipsoidal normal passing through the station and N the geoidal undulation which is the separation between the defined ellipsoid and the ideal geoid, we can write

$$H_D = H_I + c(\phi, \lambda) \tag{2}$$

where $c(\phi, \lambda)$ can be considered as a correction term, dependent on position in the datum, to correct orthometric heights referred to an ideal reference surface to the regional datum orthometric height. If sufficient points are available, the value of 'c' can be mapped and represented in some functional form depending on the size of the area and the behavior of 'c'.

Development of an height-bias model or setting procedures for mapping the bias function value 'c' in some functional form requires GPS observations (height determination specifically), precise geoid undulation model and consistent orthometric heights in a local vertical datum. Currently such information is available in the United States, Canada and in some part of Europe. On our request, the Geodetic Survey of Canada (A. Mainville) provided the GPS data for four traverses: South Alberta (106 stations), North Alberta (51 stations), Central Alberta (52 stations), and Great Slave Lake area (91 stations). In addition, the orthometric heights for all the GPS stations in the Canadian Geodetic Vertical Datum (CGVD-28) and the high resolution geoid model GSD 91 were obtained. For the United States, the National Geodetic Survey (D. Milbert) provided the data for GPS traverses in Florida (52 stations), Virginia, G105 (62 stations), and in Oregon (44 stations). They also supplied the orthometric heights for all the stations in National Geodetic Vertical Datum NGVD-29 and the high resolution Geoid Height model GEOID90 (Milbert, 1991b).

Based on the data received, efforts to develop a suitable height bias model were made during the period under report. The procedure followed is as follows:

1. Since the GPS station coordinates in the Canadian GPS traverses were given in the WGS 84 coordinate system, they were first transformed to the ITRF 90 geocentric coordinate system using the transformation parameters given in Boucher and Altamimi (1991), to make the origin of the coordinate system compatible with the geopotential model used. In the case of GPS traverses in the United States, the station coordinates were already in the ITRF 90 geocentric system as reported by Milbert (1991a) and hence no transformation was required. The geoidal undulation at these stations were then interpolated using the respective geoid models.

2. The orthometric height bias 'c' was then computed using the relation,

$$c(\phi, \lambda) = (h_{GPS} - N_{Geoid\ Model}) - H_{local\ datum} \quad (3)$$

Statistical analysis of the height bias values for US and Canadian traverses are given in Table 1. An example of an orthometric height bias table is given in Table 2 for the Slave Lake Area.

Table 1: Statistics of the Difference in Orthometric Heights Derived from GPS/GSD91 and CGVD28 Heights in case of Canadian Traverses and from GPS/GEOID90 and NGVD29 Heights for GPS Traverses in the United States. Units are in cm.

Traverse	United States				Canada			
	Oregon	Florida	Virginia G105	Tennessee	North Alberta	Central Alberta	South Alberta	Great Slave Lake Area
# of Stations	44	52	61	49	51	52	106	91
Min. Ht. Bias Value	-36	-77	-64	73	-27	-89	-27	-31
Max. Ht. Bias Value	+58	+49	+90	153	+62	-45	+84	+48
Mean	2	3	-5	119	-1	-59	23	-3
Std. Deviation	22	39	39	22	11	8	21	16

3. Three different procedures were attempted to map the height bias function value 'c'. They were:

   - global surface fitting using deterministic polynomial functions,
   - minimum curvature fitting using cubic spline functions, and
   - height bias modelling using least squares collocation technique.

Since the polynomial functions provide unpredictable errors in the interpolated values away from the stations and also the minimum curvature technique do not provide the estimated accuracy of predicted height bias function values, the least squares collocation technique was attempted for modelling the height bias function value 'c'. Assuming a correlation length of 40 km and using a second-order Markov model covariance function fitted to local data, the height bias value along with its estimated accuracy were predicted for any other location in the area using least squares collocation technique. Contour plots showing predicted height bias function in the area and its estimated accuracy one each for U.S. traverse (in Oregon) and Canadian traverse (in South Alberta) are shown in Figures 1 (a, b) and 2 (a, b) respectively with a contour interval of 5 cm. From Figures 1(b) and 2(b), we see that the estimated accuracy of predicted height bias function is of the order of 20 cm if we have well distributed GPS stations in the area. The advantage of developing such an height bias model is that, by knowing the orthometric height with respect to an ideal geoid using Geoid/GPS observations using the relation (1), its orthometric height in the local vertical datum can be computed using the interpolated bias function 'c'. Also, the plot showing the estimated accuracy of predicted 'c' values, serves as a reliability diagram intimating the user the accuracy of the predicted 'c' value and hence the orthometric height in local vertical datum.

Considering Figure 1a, one sees a clear large signature that is associated with the higher elevations in the western part of Oregon. These larger bias terms may be associated with geoid undulation errors in Geoid 90 associated with the indirect effect.

Table 2 : Computation of Orthometric height bias function in Great Slave Lake Area, Canada. Heights in meters

Lat (deg)	Lon (deg)	Ellip.ht (h)	geoid.ht (N) (from GSD91)	(h-N)	Ortho.ht (H) (in CGVD28)	c=(h-N)-H Ht.error (in cm)
62.475	245.559	183.350	-25.905	209.255	209.229	2.5
60.038	243.117	279.969	-19.840	299.809	299.902	-9.2
60.426	243.647	263.951	-21.571	285.522	285.730	-20.7
60.800	243.408	244.827	-21.935	266.762	267.028	-26.5
60.709	244.995	216.481	-24.809	241.290	241.269	2.0
62.479	245.273	155.184	-25.623	180.807	180.706	10.1
62.544	245.010	141.715	-25.083	166.798	166.700	9.8
62.593	244.813	142.511	-24.582	167.093	167.061	3.2
62.660	244.705	152.975	-24.274	177.249	177.221	2.8
62.686	244.535	148.808	-23.980	172.788	172.753	3.4
62.709	244.411	147.611	-23.776	171.387	171.339	4.8
62.784	244.007	140.531	-22.935	163.466	163.455	1.0
62.641	243.743	210.678	-22.284	232.962	232.996	-3.3
62.556	243.591	233.449	-22.033	255.482	255.507	-2.5
62.382	243.503	250.968	-21.915	272.883	272.957	-7.3
62.302	243.571	221.520	-22.063	243.583	243.699	-11.6
62.204	243.683	206.345	-22.334	228.679	228.722	-4.3
62.109	243.704	191.012	-22.422	213.434	213.524	-9.0
62.024	243.687	188.108	-22.451	210.559	210.650	-9.1
61.979	243.563	196.285	-22.232	218.517	218.608	-9.1
61.837	243.329	203.860	-21.826	225.686	225.825	-13.8
61.781	243.228	201.425	-21.666	223.091	223.148	-5.7
61.711	243.103	197.065	-21.465	218.530	218.533	-0.3
61.677	242.975	190.166	-21.261	211.427	211.571	-14.4
61.601	242.859	175.645	-21.128	196.773	196.921	-14.7
61.514	242.754	154.450	-21.025	175.475	175.646	-17.1
61.428	242.601	138.736	-20.823	159.559	159.752	-19.2
61.366	242.499	136.108	-20.732	156.840	157.021	-18.0
61.095	242.500	174.119	-20.585	194.704	194.958	-25.4
60.870	243.269	206.432	-21.749	228.181	228.496	-31.4
60.717	243.528	231.687	-22.030	253.717	253.948	-23.0
60.629	243.659	240.712	-22.095	262.807	262.961	-15.3
60.784	245.342	211.651	-25.497	237.148	237.128	2.0
60.611	245.510	245.756	-25.469	271.225	271.277	-5.2
60.536	245.610	222.953	-25.550	248.503	248.527	-2.4
60.407	245.733	242.665	-25.609	268.274	268.213	6.1
60.228	246.153	247.940	-25.958	273.898	273.778	11.9
60.174	246.295	239.781	-26.070	265.851	265.705	14.6
60.149	246.444	241.794	-26.275	268.069	267.889	18.0
60.120	246.619	232.888	-26.527	259.415	259.224	19.0
60.035	246.872	226.431	-26.810	253.241	253.042	19.9
60.026	247.027	219.796	-27.054	246.850	246.597	25.3
60.014	247.753	161.315	-28.010	189.325	188.848	47.7
60.013	247.955	172.707	-28.241	200.948	200.560	38.7
60.005	248.046	178.882	-28.335	207.217	206.868	34.8
59.999	248.162	183.599	-28.439	212.038	211.711	32.7
62.458	245.413	165.830	-25.813	191.643	191.579	6.3

62.519	245.101	160.654	-25.285	185.939	185.810	12.9
62.759	244.193	158.507	-23.318	181.825	181.830	-0.4
60.456	245.675	229.981	-25.583	255.564	255.514	4.9
60.283	245.955	245.985	-25.714	271.699	271.624	7.4
61.185	246.301	132.292	-27.566	159.858	159.663	19.4
61.313	242.400	138.238	-20.580	158.818	158.955	-13.6
60.106	243.242	269.264	-20.182	289.446	289.596	-14.9
60.173	243.308	267.289	-20.420	287.709	287.873	-16.3
60.263	243.424	269.261	-20.821	290.082	290.243	-16.0
60.343	243.555	269.673	-21.223	290.896	291.060	-16.3
60.506	243.747	251.176	-21.990	273.166	273.324	-15.8
60.625	243.939	190.459	-22.742	213.201	213.289	-8.7
60.684	244.067	170.230	-23.100	193.330	193.446	-11.6
60.739	244.482	165.580	-23.981	189.561	189.673	-11.1
60.725	244.645	167.320	-24.253	191.573	191.611	-3.8
60.740	245.179	204.241	-25.163	229.404	229.384	2.0
60.804	245.465	206.889	-25.756	232.645	232.681	-3.5
61.039	246.312	135.807	-27.525	163.332	163.270	6.1
60.838	244.220	138.667	-23.710	162.377	162.663	-28.6
60.000	247.395	197.123	-27.534	224.657	224.418	23.8
60.047	247.190	203.371	-27.345	230.716	230.463	25.2
60.046	246.745	242.739	-26.620	269.359	269.141	21.7
60.329	245.851	241.294	-25.623	266.917	266.865	5.2
60.670	245.404	232.053	-25.382	257.435	257.380	5.5
60.698	245.256	230.009	-25.197	255.206	255.157	4.8
60.825	245.629	190.050	-26.062	216.112	216.110	0.2
60.865	245.759	179.217	-26.381	205.598	205.594	0.4
60.928	245.883	165.257	-26.669	191.926	191.923	0.2
60.986	246.180	139.620	-27.235	166.855	166.851	0.4
61.132	246.370	131.929	-27.653	159.582	159.459	12.3
60.965	246.045	137.190	-27.013	164.203	164.180	2.346
60.731	244.844	178.432	-24.599	203.031	203.107	-7.581
60.742	244.260	151.468	-23.571	175.039	175.221	-18.233
60.766	244.148	148.883	-23.434	172.317	172.526	-20.886
60.559	243.860	236.930	-22.382	259.312	259.488	-17.575
60.001	243.018	276.084	-19.586	295.670	295.814	-14.376
60.985	242.753	171.685	-20.924	192.609	192.884	-27.462
60.931	243.082	226.397	-21.475	247.872	248.167	-29.489
60.936	242.933	214.140	-21.195	235.335	235.559	-22.448
61.048	242.599	187.730	-20.697	208.427	208.678	-25.071
61.173	242.469	152.332	-20.634	172.966	173.093	-12.733
61.255	242.473	134.933	-20.709	155.642	155.604	-16.164
61.731	243.139	208.827	-21.516	230.343	230.492	-14.921
60.654	243.634	243.395	-22.101	265.496	265.708	-21.137

FIGURE 1A
HEIGHT BIAS FUNCTION WITH THE OREGON DATA SET

CI = 5 cm

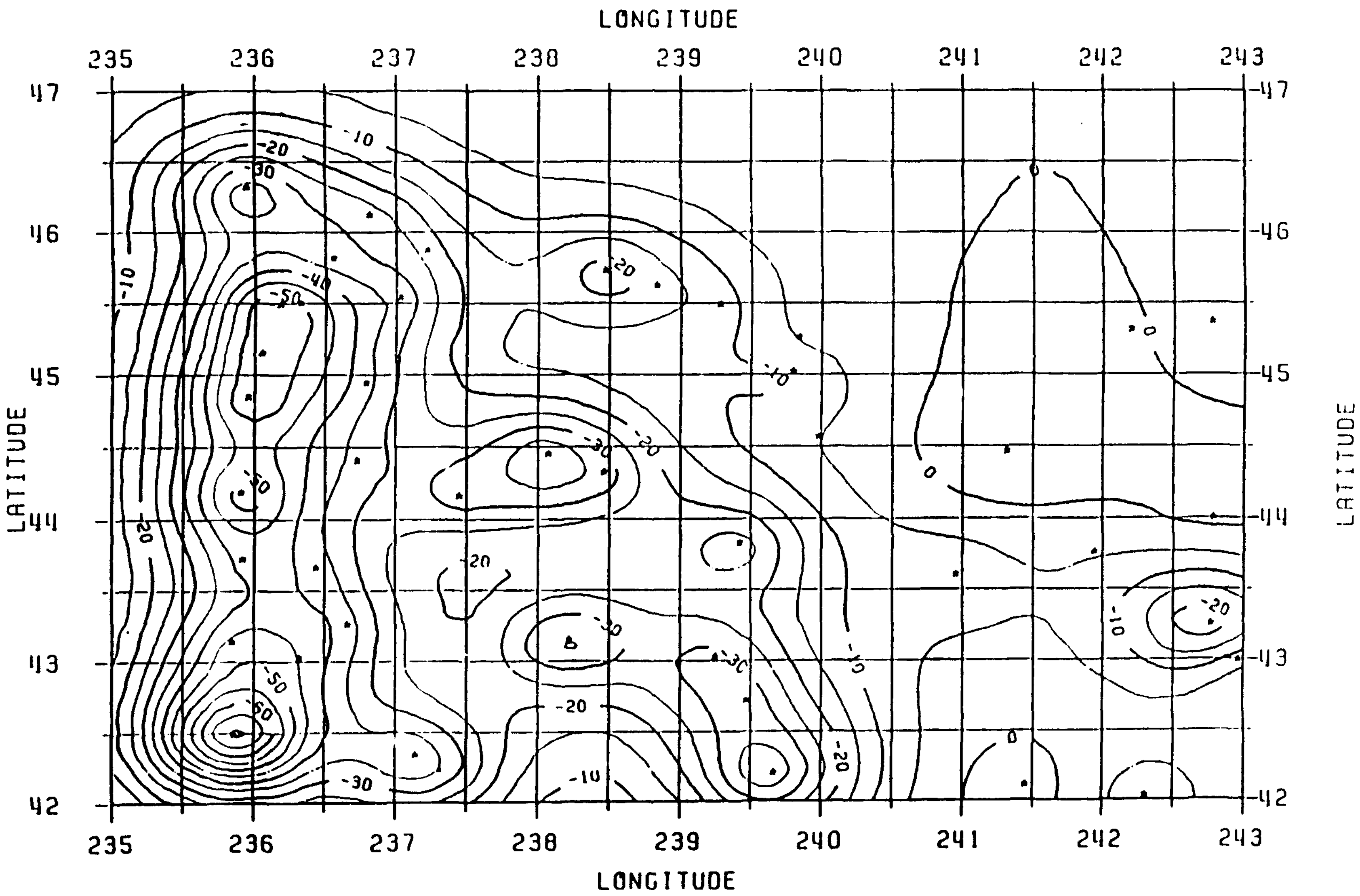


FIGURE 1B
ACCURACY OF ESTIMATE OF PREDICTED HEIGHT BIAS FUNCTION WITH THE OREGON DATA SET

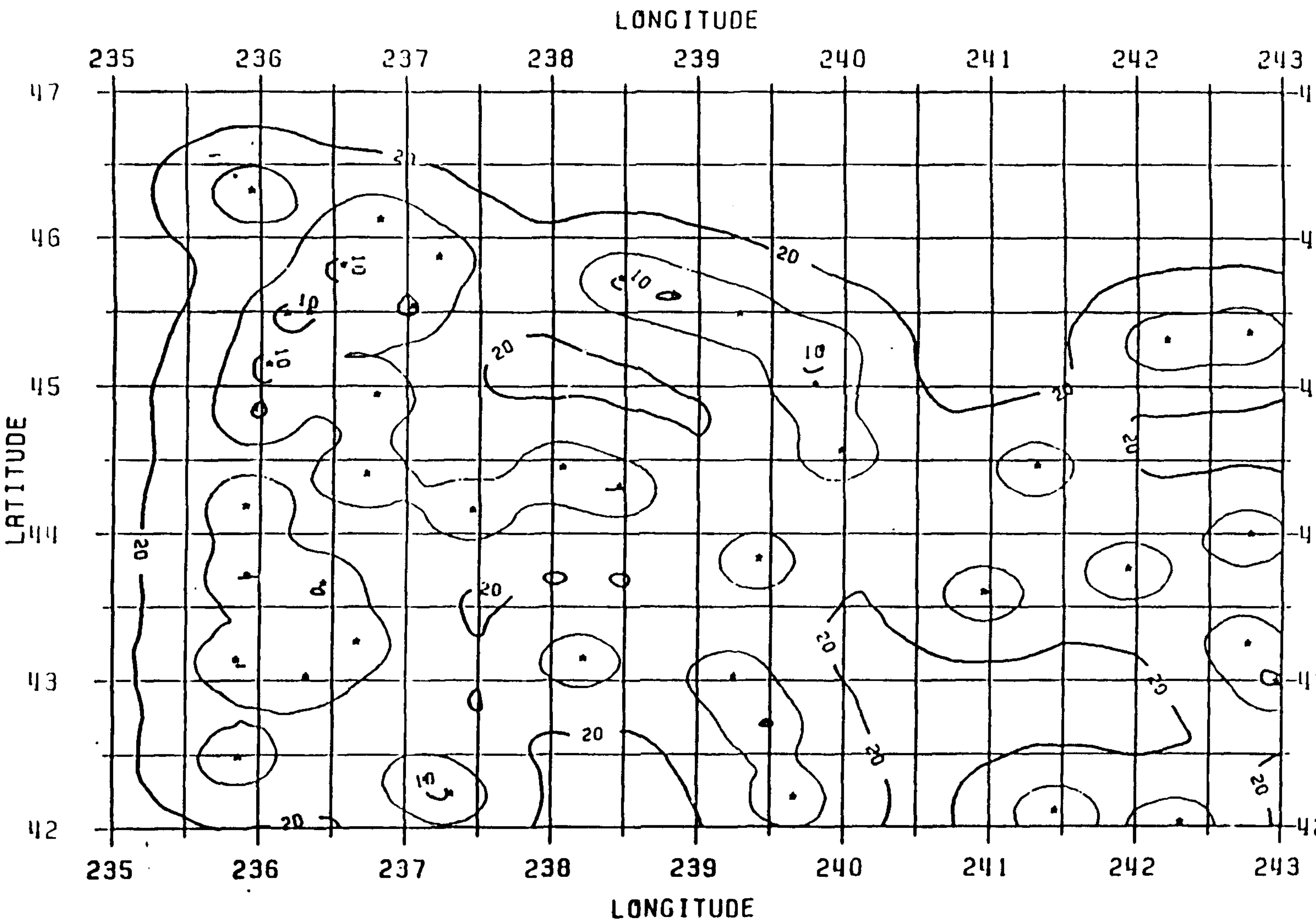


FIGURE 2A
HEIGHT BIAS FUNCTION WITH THE SOUTH ALBERTA DATA SET

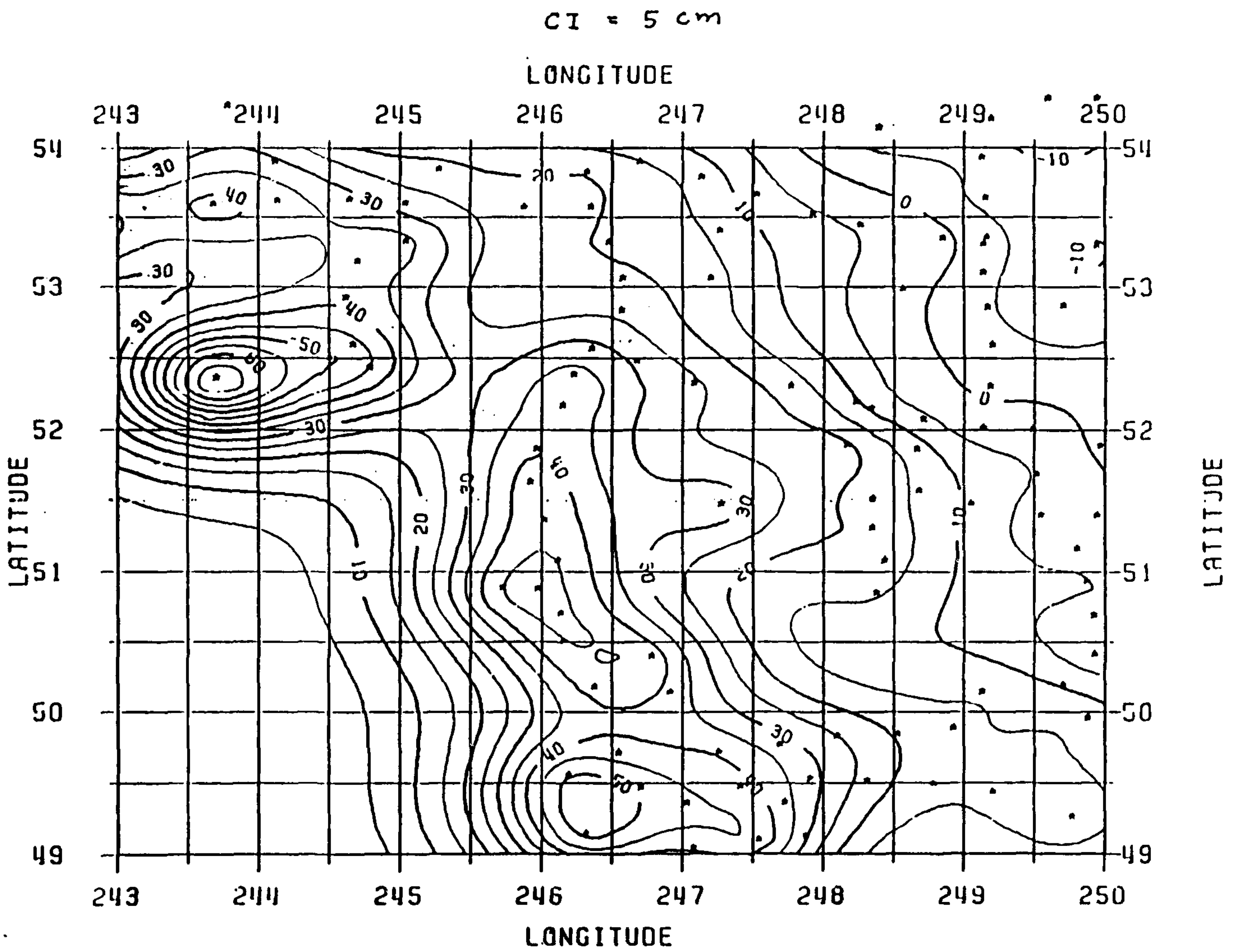
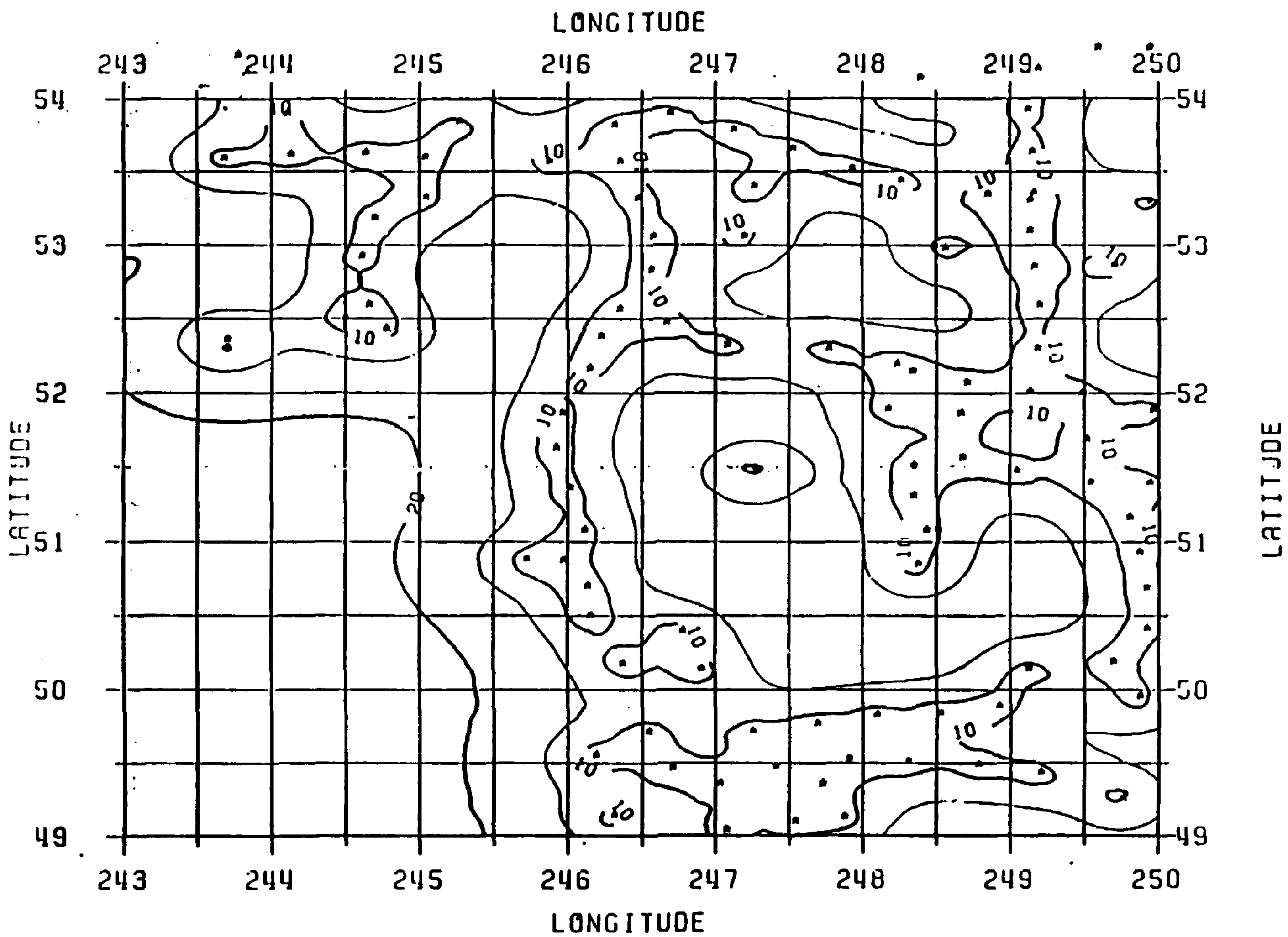


FIGURE 2B
ACCURACY ESTIMATE OF PREDICTED HEIGHT BIAS FUNCTION WITH THE SOUTH ALBERTA DATA SET

cI = 5 cm



2.1.2 Studies in the World Height System Definition

Basically four kinds of data are essential for modelling the World Height System (WHS). They are: accurately determined free-air gravity anomalies, precise orthometric heights of the stations above the regional vertical reference datums, ellipsoidal heights of the stations above adopted goecentric ellipsoid and accurate geopotential models. Since such data availability over the world are neither uniform nor to the required precision, different modelling procedures are needed to be developed to accommodate the available heterogeneous data to come up with an elegant observation equation for estimating the parameters defining a WHS. As understandable, it may require several iterations before realizing an ideal WHS which can meet the present day requirements of WHS established to an accuracy of ±10 cm.

The two different data scenarios anticipated in developing the procedure for setting up the observation equations for defining the WHS are:

(a) Given the free-anomalies in the Molodensky sense and normal heights of the stations, and
(b) Given the free-air anomalies in the classical sense and orthometric heights (or normal orthometric heights) of the stations.

Detailed discussion on the definition and comparison of free-air anomalies obtained in Molodensky sense and Classical sense can be found in Sideris and Forsberg (1991) and for various height definitions given in Rapp and Balasubramania (1992).

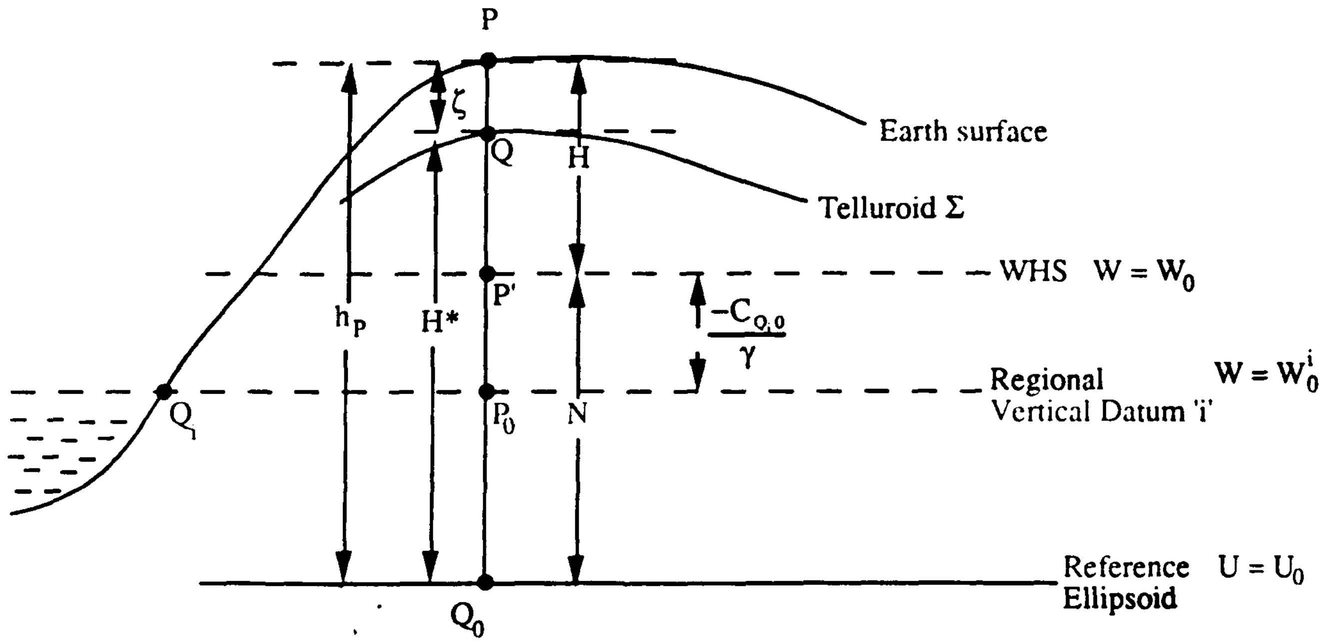


Fig. 3 Geometric representation of various equipotential surfaces

In Figure 3, Q_i is the fundamental benchmark serving as the origin for the regional vertical datum 'i'. The potential of the reference surface for WHS is W_0. We define $C_{Q_i0} = W_0 - W_0^i$ as the potential difference between the reference equipotential surfaces for datum 'i' and the WHS reference surface. Introducing a reference ellipsoid where U_0 is the potential on the surface, we define $\Delta W_0 = W_0 - U_0$. The distance between the equipotential

surfaces $U = U_0$ and $W = W_0$ is also shown in the figure, and depends on the disturbing potential at P' and ΔW_0.

Procedure for setting up the observation equation for case (a) is as follows:

Let H^* be the normal height of the point P with respect to the WHS (to be defined) and H_i^* the known normal height of the same point referred to i^{th} regional vertical datum. Then from equation (2-122) in Pavlis (1988), we can write the relation:

$$\Delta g_M \approx \Delta g_M^{(i)} - \frac{1}{\gamma_{Q_0}}\left[\left(\frac{\partial\gamma}{\partial h}\right)_{Q_0} + \left(\frac{\partial^2\gamma}{\partial h^2}\right)_{Q_0} \cdot H_i^*\right] \cdot C_{Q_i0} \tag{4}$$

where $\Delta g_M, \Delta g_M^{(i)}$ are the gravity anomalies computed with normal gravity γ_Q determined using normal heights H^*, H_i^* of the point P referring to WHS and regional vertical datum 'i' respectively.

From Moritz (1980, eq. 48-1), the computation of height anomaly ζ_P by analytical continuation to point level can be done using the formula:

$$\zeta_P = \zeta_0 + \frac{R}{4\pi\gamma}\iint\left(\Delta g_M^C + g_1\right)S(\psi)d\sigma \tag{5}$$

where $\zeta_0 = \frac{\delta GM}{R\gamma} - \frac{\Delta W_0}{\gamma}$

Δg_M^C is the free-air anomaly defined in Molodensky sense and corrected for systematic effects as discussed in Rapp and Pavlis (1990),

g_1 is the Molodensky correction term,

R is the mean radius of earth, γ the mean gravity, and

$S(\psi)$ the Stokes' function.

If Faye anomalies are available equation (5) can be replaced with eqn. (24) in Wang (1993) as:

$$\zeta_P = \zeta_0 + \frac{R}{4\pi\gamma}\iint\left(\Delta g_M^C + C\right)S(\psi)d\sigma - \frac{\Delta g_B}{\gamma} - \frac{\pi G\rho}{\gamma}H_P^{*2} - \frac{\pi G\rho}{\gamma}\delta h^2 \tag{6}$$

where H_P^* is the normal height of point P and

$\delta h^2 = 0.453 - 0.018 \sin\phi + 0.087 \cos\phi\cos\lambda + 0.204 \cos\phi\cos\lambda$ (km^2)

Combining equations (4) and (6), we get

$$\zeta_P = \zeta_0 + \frac{R}{4\pi\gamma}\iint\left(\Delta g_M^{c^{(i)}} + C\right)S(\psi)d\sigma - \frac{R}{4\pi\gamma}\sum_{j=1}^{n} q^j C_{Q_j0}\int_{\sigma_j} S(\psi)d\sigma_j$$
$$-\frac{\Delta g_B}{\gamma}H_i^* - \frac{\pi G\rho}{\gamma}H_i^{*2} - \frac{\pi G\rho}{\gamma}\delta h^2 \tag{7}$$

where $q^j = \frac{1}{\gamma_{Q_0}}\left[\left(\frac{\partial\gamma}{\partial h}\right)_{Q_0} + \left(\frac{\partial^2\gamma}{\partial h^2}\right)_{Q_0} H_j^*\right]$

From equation (2-121) and (2-122) in Heiskanen and Moritz (1967) we can write

$$q^j \approx -\frac{2}{R} \tag{8}$$

From Figure 3 we note that:

$$H_P^* = H_i^* + \frac{C_{Q_i0}}{\gamma} \text{ and } h_P = H_P^* + \zeta_P \text{ or } \zeta_P = h_P - H_P^* \tag{9}$$

where h_P is the ellipsoidal height of the point P and ζ_P its height anomaly. Comparing eqns. (7) and (9) and rearranging known and unknown terms we write:

$$Y = h - H_i^* - \zeta_i^i = \frac{-\Delta W_0}{\gamma} + \frac{C_{Q_i0}}{\gamma} + \frac{2}{\gamma}\sum_{j=1}^{n} C_{Q_j0}\left\{\frac{1}{4\pi}\int_{\sigma_j} S(\psi)d\sigma_j\right\} \tag{10}$$

where $\zeta_i^i = \frac{R}{4\pi\gamma}\iint_\sigma\left(\Delta g_M^{c^{(i)}} + C\right)S(\psi)d\sigma - \frac{\Delta g_B}{\gamma}H_i^* - \frac{\pi G\rho}{\gamma}H_i^{*2} - \frac{\pi G\rho}{\gamma}\delta h^2$ is the gravimetric height anomaly (without ζ_0 term) and referred to regional vertical datum 'i'.

Similarly for case (b) when the free-air anomalies in classical sense are given along with orthometric heights of stations, modelling procedure given in Rapp and Balasubramania (1992) can be followed to come up with the observation equation:

$$Y = h - H_i - N_i^i = \frac{-\Delta W_0}{\gamma} + \frac{C_{Q_i0}}{\gamma} + \frac{2}{\gamma}\sum_{j=1}^{n} C_{Q_j0}\left\{\frac{1}{4\pi}\int_{\sigma_j} S(\psi)d\sigma_j\right\} \tag{11}$$

where $N_i^i = \frac{R}{4\pi\gamma}\iint\left(\Delta g_c^c + C\right)S(\psi)d\sigma - \frac{\pi G\rho}{\gamma}H_i^2$ is the gravimetric geoid undulation (without N_0 term) and referred to regional datum 'i'.

Equations (10) and (11) are the key observation equations for solving the unknown parameters ΔW_o and C_{Q_i0} (for i = 1, 2, ... I). The parameters can be obtained using a least-squares adjustment procedures. The adjustment process discussed by Rapp and Balasubramania (1992, Section 4.4) can be used to estimate the parameters defining the World Height System.

The solution of the parameter vector is given by

$$\hat{X} = \left(A^T \Sigma_y^{-1} A + K^T K\right)^{-1} A^T \Sigma_y^{-1} Y \tag{12}$$

and the error covariance matrix of the estimated parameters would be

$$\Sigma_{\hat{X}} = \left(A^T \Sigma_y^{-1} A + K^T K\right)^{-1} A^T \Sigma_y^{-1} A \left(A^T \Sigma_y^{-1} A + K^T K\right)^{-1} \tag{13}$$

where Y is the nx1 observation vector from equation (10),
A is the nxm coefficient matrix of observation,
X is the mx1 parameter vector,
K is the 1xm coefficient matrix of the constraints,
Σ_y^{-1} is the variance-covariance matrix of observations.

Equation (12) and (13) correspond to equations (4-33) and (4-34) in Rapp and Balasubramania (ibid.).

Numerical Investigation

A first iteration test computation to estimate the parameters defining the WHS was carried out with available data from six regional vertical datums. The different vertical datums considered are: North American Vertical Datum 88 (NAVD88), ODN (UK), IGN69 (France), NN (Germany), AHD71 (Australia) and Scandinavian datum. Though there is no single regional vertical datum known as the Scandinavian datum, the individual datums RH70 (Sweden), N60 (Finland) and NN1954, NNN1957 (Norway) were combined to serve as a single regional datum for computational convenience.

The ITRF91 coordinate system was selected as the consistent coordinate system for defining the space geodetic station coordinates used in the realization of WHS. An ideal ellipsoid with the following parameters was adopted as the reference ellipsoid for the study:

$a_e = 6378136.3$ m
$f = 1/298.257222101$
$GM = 3986006 \times 10^8 \ m^2 s^{-2}$
$\omega = 7292115 \times 10^{-11}$ rad S^{-1}

The last three parameters are the same as the parameters of the GRS80 reference ellipsoid.

Table 3 below lists the distribution of space stations in various regional vertical datums and Figure 4 shows their locations.

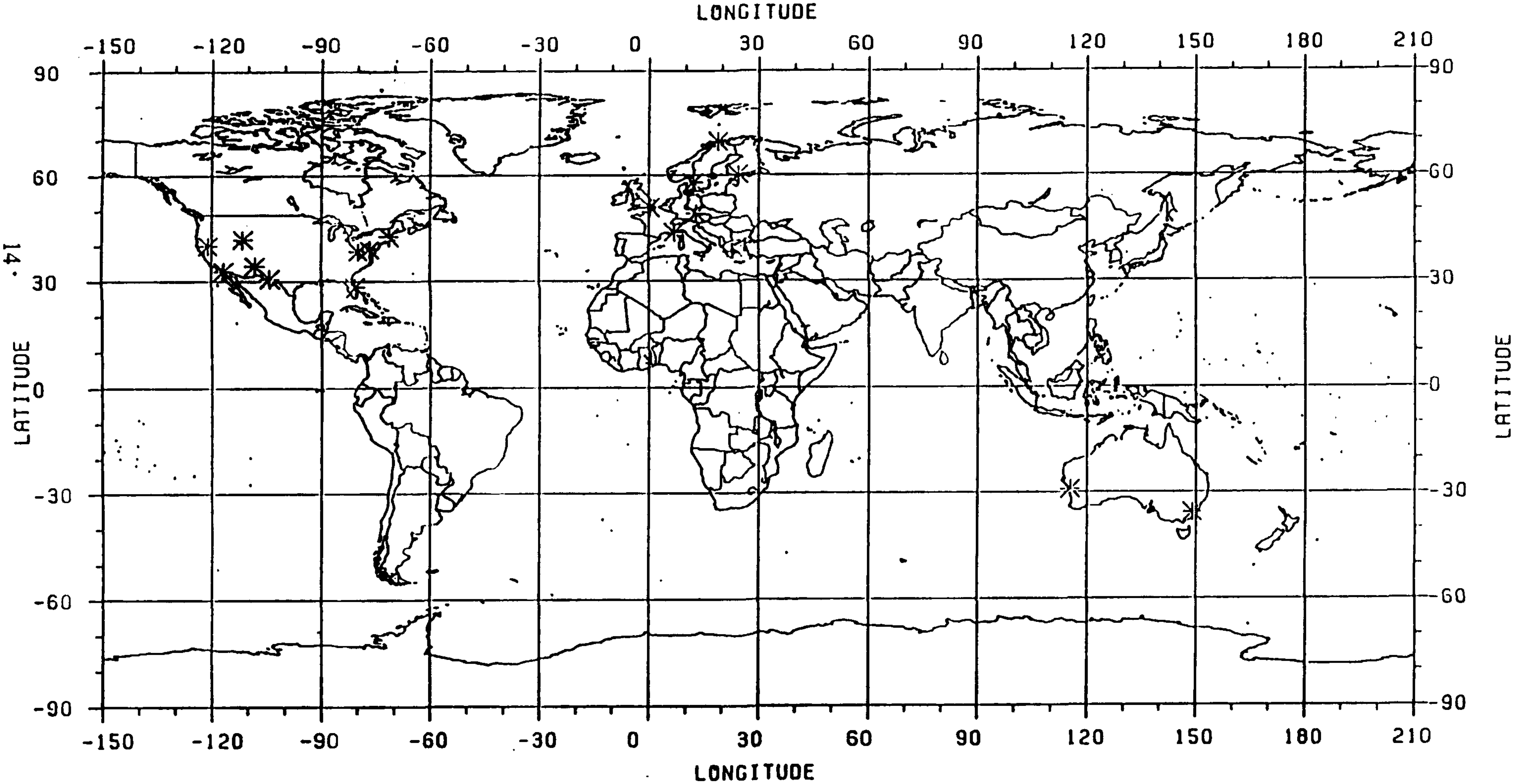
Table 3: Distribution of space stations

DATUM	NAVD88	Scandinavian	NN	IGN69	ODN	AHD71
No. of Stations	10	3	1	1	1	2

The accuracy and grid size of terrestrial gravity data used varied from one regional datum to another. For North American datum, 3′x3′ atmospherically connected Helmert anomalies around each of the U.S. stations received from D. Milbert, NGS were used. As quoted by Milbert (1991) these 3′x3′ mean anomalies have a random error of ±1.5 mgal. For the Scandinavian datum, Faye anomalies received from Dr. René Forsberg were used. For the other stations in England, Germany, France and Australia, the 6′x10′ and 2′x2′ atmospherically

FIGURE 4

STATIONS USED IN VERTICAL DATUM DEFINITION



corrected free-air anomalies referring to OSU GRS (a = 6378136.0, f = 1/298.257222101) used by Despotakis (1983) in his thesis work were used. These gravity data sets are not corrected for terrain and as given in Torgé (1983), they have a random error of ±7 mgal.

The elevation of the space stations above their local vertical reference datum were obtained from NASA space geodesy program - catalogue of site information (NASA Tech Memo 4482). For the stations in USA, their heights referred to NGVD29 height system. Using the VERTCON software received from Zilskoski at NGS, the heights were converted to NAVD88 heights. For stations in England and Australia, the heights given were normal orthometric heights and for stations in Germany, France and Scandinavian countries, they are normal heights referred to their regional vertical datum. The global geopotential model OSU91A to degree 360 was used as the reference field in the study.

Three different techniques were used to compute the gravimetric height anomaly ζ_i^i and the gravimetric undulation N_i^i in equations (10) and (11) respectively. They were:

(i) Modified Stokes' technique
(ii) Least squares collocation technique (LSC)
(iii) Interpolation from regional geoid height models developed using Fast-Fourier transform technique.

Comparison of gravimetric undulations computed using different techniques at the U.S. space geodetic stations are given in Table 4. The interpolated geoid height values from recent GEOID93 Geoid Height model received from Milbert at NGS, are also included for better comparison. The differences between the gravimetric undulation and geometric undulation are also shown. In Table 5, gravimetric height anomaly/undulation computed at the different European space stations are given. Since we had only the regional geoid height model NKG-89 for Scandinavian countries, only for stations in that area, the interpolated geoid height are included in Table 5. The gravimetric undulation at the Australian stations computed using the first two techniques and their comparison with geometric undulation are given in Table 6.

From Tables 4 and 5 we see that the rms difference between the geometric undulation and the gravimetric undulation computed using different techniques are almost the same but in Table 6 we see they differ by about 60 cm. Also from Table 6 we note the difference between the gravimetric and geometric undulations computed at the Yarragadee station (located in Western Australia) and at Canberra (located in Eastern Australia) differ by about 1.1 meters though both the stations are considered to lie in the same regional vertical datum (AHD71). This difference may be due to the suggested rise of mean sea level up the East Australian coast of about 1.5 m by Morgan (1992) and/or the same extent due to the long wave length error in the global geopotential model used.

Table 4: Comparison of gravimetric undulation computed using different techniques at the U.S. space geodetic stations. Units are meters.

	Undulations				Differences		
Station	LSC Technique NL	Stokesian Approach NS	GEOID93 Model N93	Geometric Undulation N88	N88 - NL	N88 - NS	N88-N93
7051 Quincy	-22.796	-22.938	-23.303	-23.734	-0.938	-0.796	-0.431
7062 San Diego	-34.112	-33.635	-33.311	-33.451	0.661	0.184	-0.140
7082 Bear Lake	-13.733	-14.803	-13.944	-14.095	-0.362	-0.947	-0.151
7091 Westford	-28.768	-28.429	-28.166	-28.296	0.472	0.133	-0.130
7086 Ft. Davis	-21.632	-21.641	-21.465	-21.642	-0.010	-0.001	-0.177
7105 GSFC5	-33.089	-32.934	-32.740	-32.940	0.149	-0.006	-0.200
7110 Monument P	-32.064	-31.488	-31.685	-31.816	0.248	-0.328	-0.131
7234 Pietown	-21.127	-21.460	-21.043	-21.569	-0.442	-0.109	-0.526
7069 Patrick AF	-29.415	-29.430	-29.157	-28.813	0.602	0.617	0.344
7204 Green Bank	-31.814	-31.092	-31.124	-31.442	0.372	-0.350	-0.318
				Mean	0.075	-0.160	-0.254
				Std. Deviation	0.489	0.441	0.134
				rms	0.495	0.467	0.288

Table 5: Comparison of gravimetric height anomaly computed using different techniques at the European space geodetic stations. Units are meters.

Station	LSC technique ζ_{LSC}	Stokesian ζ_{STOKES}	NKG-89 Model ζ_{89}	Geometric height anomaly ζ_{GEOM}	$\zeta_{GEOM} - \zeta_{LSC}$	$\zeta_{GEOM} - \zeta_{STOKES}$	Remarks
7601 Metsahovi	19.603	19.388	18.431	19.751	0.148	0.363	Scandinavian Datum
1001 Onsala	35.938	35.854	35.359	37.223	1.285	1.369	
7602 Tromso	30.977	30.799	31.484	32.289	1.312	1.490	
7834 Wetzell	46.699	46.764	-	47.556	0.857	0.792	DHNN
7835 Grasse	51.203	51.932	-	52.120	0.917	0.188	IGN69
7840 RGO-UK	45.360	45.545	-	46.365	1.005	0.820	ODN
				Mean	0.819	0.771	
				Std. Deviation	0.436	0.469	
				rms	0.928	0.903	

Table 6: Comparison of gravimetric undulation computed using different techniques at the Australian space geodetic stations. Units are meters.

Station	LSC Technique N_L	Stokesian approach N_S	Geometric undulation N_G	OSU91A Model	$N_G - N_L$	$N_G - N_S$
7090 Yarragadee	-24.995	-25.337	-24.517	-24.883	0.438	0.820
7943 Canberra	18.727	18.090	20.105	18.957	1.378	2.015
				Mean	0.908	1.418
				Std. Deviation	0.470	0.598
				rms	1.045	2.366

Using the data at 18 space geodetic stations in 6 regional vertical datums, and with 7 unknowns, observation equations and normal equations were formed. From equations (12) and (13) the parameters defining the WHS and their accuracy estimates were computed for different techniques used in computing gravimetric height anomaly/undulation. The estimated parameters are given in Figures 5 and 6.

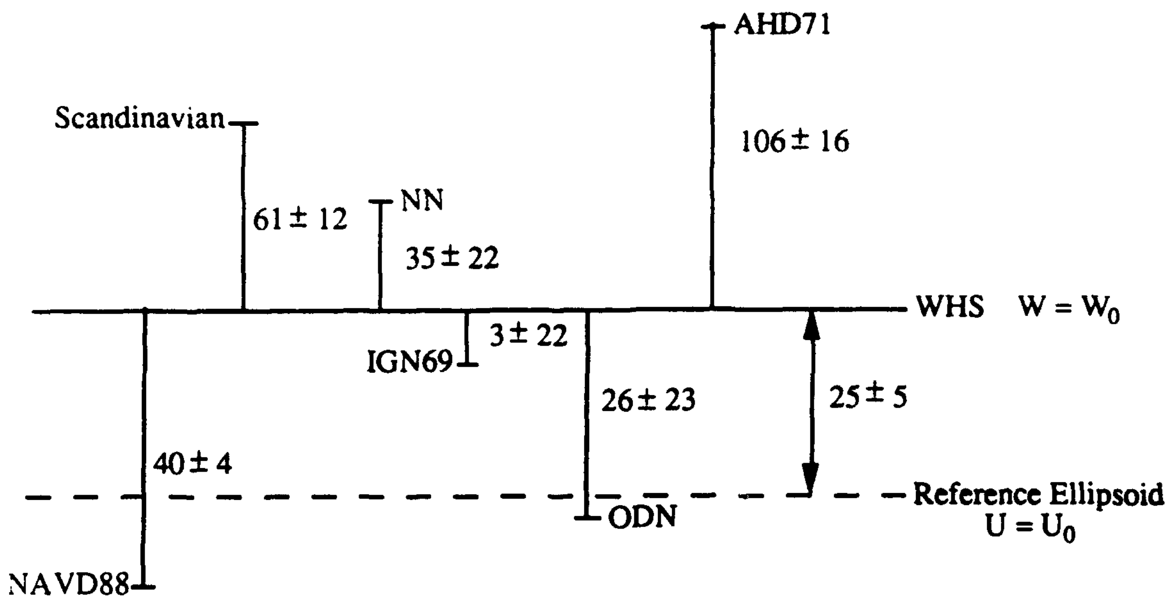


Figure 5: Regional Vertical Datum separation from defined WHS (units in cm) (from Modified Stokes' technique).

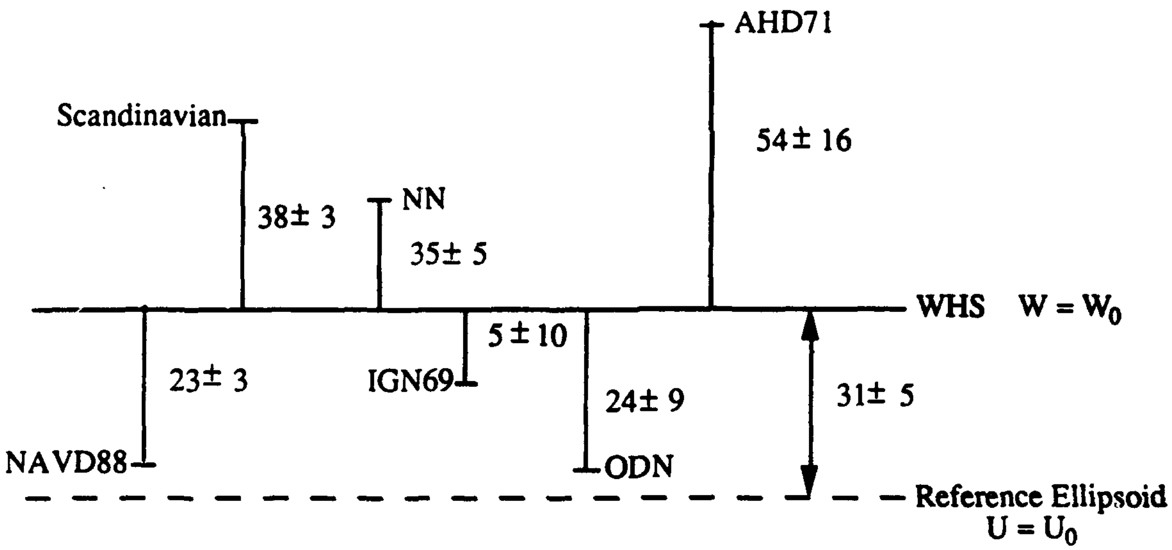


Figure 6: Regional Vertical Datum separation from defined WHS (units in cm) (from LSC technique)

From Figures 5 and 6 it is clear that the different regional vertical surfaces used in the definition of WHS show the same pattern of their relative positions to one another in spite of using two different techniques in their estimation but the numerical values are different. The Scandinavian datum which is 61 cm above the defined WHS in Figure 6 is only 38 cm above the same in Figure 6. This may be due to using limited terrestrial gravity data ($\psi = 1°$) for computing the gravimetric height anomaly at the Scandinavian stations. The Australian height datum AHD71 which is about 106 cm above the defined WHS in Figure 5 is only 54 cm in Figure 6. The reason is that gravimetric undulation computed at the Australian space station using Modified Stokes' technique and LSC technique differs by about 70 cm. This may be due to

error in the gravity anomaly data used around these stations. The datum differences between DHNN and IGN69 as intimated by Boucher (private communication) is about 50 cm which agrees well with the differences shown in Figure 5 and 6. Except for the Australian Height datum, the differences between different regional datums as shown in Figure 5 compare well with the estimates given by Rapp (1993).

2.2 Research Activities of R.H. Rapp

During this grant Professor Rapp utilized existing Doppler positioned stations to study several vertical datums with the aid of gravimetric geoid undulations derived from a merger of new JGM-2 potential coefficient model (degrees 2-70) and the OSU91A model (degrees 71 to 360). This research led to a paper that has been accepted for publication in the Bulletin Geodesique or manuscripta geodaetica. This paper is made part of this report and starts on the following page.

2.2.1 Separation Between Reference Surfaces of Selected Vertical Datums

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November 1993

Abstract

This paper discusses the separation between the reference surface of several vertical datums and the geoid. The data used includes a set of Doppler positioned stations, transformation parameters to convert the Doppler positions to ITRF90, and a potential coefficient model composed of the JGM-2 (NASA model) from degree 2 to 70 plus the OSU91A model from degree 71 to 360. The basic method of analysis is the comparison of a geometric geoid undulation derived from an ellipsoidal height and an orthometric height with the undulation computed from the potential coefficient model The mean difference can imply a bias of the datum reference surface with respect to the geoid. Vertical datums in the following countries were considered: England, Germany, United States, and Australia. The following numbers represent the bias values of each datum after adopting an equatorial radius of 6378136.3m: England (-87 cm), Germany (4 cm), United States (NGVD29 (-26 cm)), NAVD88 (-72 cm), Australia AHD (mainland, -68 cm); AHD (Tasmania, -98 cm). A negative sign indicates the datum reference surface is below the geoid. The 91 cm difference between the datums in England and Germany has been independently estimated as 80 cm. The 30 cm difference between AHD (mainland) and AHD (Tasmania) has been independently estimated as 40 cm. These bias values have been estimated from data where the geometric/ gravimetric geoid undulation difference standard deviation, at one station, is typically ±100 cm, although the mean difference is determined more accurately.

The results of this paper can be improved and expanded with more accurate geocentric station positions, more accurate and consistent heights with respect to the local vertical datum, and a more accurate gravity field for the Earth. The ideas developed here provide insight on the determination of a world height system.

Introduction

This paper is written to estimate the linear separation between the origin reference surface of several vertical datums. The process for doing this inherently implies a concept of a world height system as discussed by Rapp and Balasubramania (1992). This paper is not meant to be an extensive analysis of theoretical procedures involved in vertical datum definition and connection. Recent papers in this area include those by Xu and Rummel (1991) and Heck and Rummel (1990). This paper is not meant to show results of the latest measurement procedures. Examples of such papers are those of Rizos, Coleman and Ananga (1991) and Pan and Sjöberg (1993). This paper is designed to examine some earlier ideas (Rapp, 1983) using the latest gravity field models and terrestrial reference frame transformations.

Principles

Let N be the geoid undulation referred to a geocentric reference frame, an ellipsoid of a specified flattening, and an equatorial radius considered to be an optimum estimate. (Optimum may be a matter of opinion so this term is loosely used here.) Our calculation of N will be done through a potential coefficient model of degree L.

Given the fully normalized coefficients C_{nm}, S_{nm}, one has

$$N(r,\theta,\lambda) = \frac{GM}{\gamma r} \sum_{n=2}^{L} \left(\frac{a}{r}\right)^n \cdot \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm}(\cos\theta) + c \quad (1)$$

where the usual definitions (Rapp and Pavlis, 1989) apply. The c term represents the downward continuation effects from an evaluation of the series at a valid point in space to a point on the geoid. We will assume that c is zero in this analysis. The even degree zonal harmonics in (1) refer to the adopted reference ellipsoid.

Let h be the ellipsoidal height of a point computed from rectangular coordinates in a geocentric reference frame in a true scale system. Let H be the orthometric height with respect to the geoid that is described by the N values of eq. (1). Then:

$$h = H + N \quad (2)$$

If one were given h and N then H can be computed. The accuracy of the determination of this H will depend on the accuracy of h, N and the optimum equatorial radius.

Consider next an orthometric height (H_D) referred to a specific vertical datum D. The ideal situation is that the vertical datum is defined by an equipotential surface, near mean sea level, at one specific point. In practice, vertical datums may have been defined by fixing mean sea level to zero orthometric elevation. In this case, the reference surface and the H values have some distortion with respect to the ideal case. In this paper, we will assume that a vertical datum is defined by a unique reference surface. This surface may not and actually will not coincide with the ideal reference surface, the geoid.

Let B be the bias between the ideal system and a specific system such that:

$$H = H_D + B \quad (3)$$

where H is the orthometric height in the ideal system and H_D is the corresponding height in the datum system. A positive B indicates the specific datum reference surface is above the ideal reference surface. We can estimate B by substituting (3) into (2):

$$B = (h - H_D) - N \quad (4)$$

The evaluation of B should be done using numerous stations connected to the datum D. Averaging individual values can reduce the noise inherent in the estimate of the three quantities on the right hand side of eq. (4). A comparison of B from different datums will yield information on the relative positions of these vertical datums.

Data

The stations to be analyzed for this paper have been positioned through Doppler observation techniques during the time period 1971-1986. A description of some of the processing techniques used in the estimation of the precise positions of an analogous station set may be found in Weigel (1993, p. 14). With this station set, "heights" above mean sea level are generally available. These heights are not precisely defined in terms of datum reference, type of orthometric height, or if in fact they may be normal heights. Although definitive results of this type of investigation require such information, this paper is considered a demonstration study and we will assume that the mean sea level heights are orthometric heights connected to the vertical datum of the region or country. Ultimately, we will restrict our analysis to stations that can be associated with a specific vertical datum.

Before any analysis is carried out, the ellipsoidal height given in the original data record is transferred to the ITRF90 system (Boucher and Altamimi, 1992, Table 4 (NWL 9D)) and referred to an ellipsoid whose equatorial radius is 6378136.3m and flattening is 1/298.257.

Analyses such as these are made more feasible as the knowledge of the Earth's gravity field improves. One of the recent models is JGM-2 (Lerch et al., 1993) which is a combination model complete to degree 70. This model has been augmented by the OSU91A model (Rapp, Wang, Pavlis, 1991) from degree 71 to degree 360. Although all tests to be described in this paper have been carried out with both the OSU91A and JGM-2 (augmented) model only (with one exception) results with the most current model (JGM-2) will be given. The geoid undulation can be computed from this data with an estimated accuracy of ±57 cm (Rapp, Wang, Pavlis, p. 64, ibid.) globally, which may be considerably poorer in areas lacking adequate gravity information.

Results - Global

The first computation involved the examination of all Doppler positions independent of their geographic location. Undulation comparisons were made with both the OSU91A and JGM-2 (augmented) potential coefficient models. In this analysis, stations with residuals exceeding 2m in absolute value were rejected from the statistical computations. This 2m criteria is

tighter than the 4m criteria used in past studies, but it is consistent with the criteria adopted for the regional studies to be reported shortly. The 2m criteria is basically twice the standard deviation of the geometric/gravimetric difference and thus is a 2 sigma criteria. The statistics on the differences are given in Table 1 where the mean difference is defined through the geometric undulation minus the potential coefficient implied undulation. The total number of stations considered was 2033. The results indicate little preference for one geopotential model although the use of the 91A model allows 11 more stations to be considered. If the comparisons were made being more selective on Doppler stations to be used (at least 35 passes are observed after June 1977) the mean differences given in Table 1 become more positive by 15 cm and the standard deviation decreases to ±87 cm. Assuming, somewhat incorrectly, that the undulation differences are independent, the standard deviation of the mean difference is approximately ±3 cm.

Table 1

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for a Global Set of Doppler Stations

	Geopotential Model	
	OSU91A	IGM-2/OSU91A
Mean Diff.	-9 cm	-15 cm
SD Diff.	±100 cm	99 cm
No. Stat. Accepted	1428	1417

Results - England/Germany

The next step was to examine stations that could be closely associated with two regions with different vertical datums. The first area was England with stations selected in the region 50° to 60° N, 355° to 2°E. We assume that the orthometric heights we have are given in the Newlyn Datum (Ashkenazi et al., 1990). The second area was Germany with stations selected in the region 48° to 54° N, 6° to 13°E. We assume that the orthometric heights we have refer to the vertical datum described in Torge (1980, Section 6.2.3).

The results of the comparisons, based on the 2m rejection criteria, are given in Table 2. The mean differences imply that the Newlyn Datum is -87 cm below the geoid while the German datum is 4 cm above the geoid. The difference implies that the German vertical datum reference surface is 91 cm above the Newlyn Datum.

Table 2

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in England and Germany

	England	Germany
Mean Diff.	-87 cm	4 cm
SD Diff.	±64 cm	±75 cm
No. Stat. Accepted	29	28

An independent estimate of the above can be obtained as follows. Willis et al (1989) estimate (on the basis of several sources of information) that the origin of the French vertical datum (IGN69) is 30 cm ±8 cm above the origin surface of the Newlyn Datum (ODN). On the basis of the comparison of heights near the French/German border (Boucher 1993, private communication), the German system origin is 50 cm above the IGN69 system (i.e. H (Germany) = H (IGN69) - 50 cm). This numerical value can be caused not only by vertical datum differences, but also differences in normal heights and orthometric heights. The latter we will ignore for this paper. Taking the 30 and 50 cm into account would imply that the German origin surface is 80 cm above the Newlyn Datum surface. This number can be contrasted with the 91 cm obtained through the Doppler station analysis.

Results in the United States

The orthometric heights for the Doppler stations in the United States are given with respect to the National Geodetic Vertical Datum of 1929 (NGDV29). This datum was defined through a general adjustment in 1929 with height constraints imposed at 26 tide gauge stations. The orthometric heights were computed using normal corrections since no gravity observations were available along the leveling lines.

A new adjustment of leveling data has led to the North American Vertical Datum of 1988 (Zilkoski, Richards, and Young, 1992). This datum is based on geopotential numbers fixing the height at a primary tidal benchmark. Although we did not have heights for the Doppler stations directly in the NAVD88 system, a datum conversion software program (VERTCON, version 1.0, February 1993) has been developed at the National Geodetic Survey. This software was used to convert the NGVD29 heights into NAVD88 heights using a transformation procedure in lieu of the direct adjustment of the points in NAVD88.

The stations to be used in this analysis were initially selected in the region: 30° to 45°N, 240° to 290°E. This station set was then thinned to eliminate stations geographically close so that a file containing 350

stations was analyzed. The results of the undulation comparison with the JGM-2/OSU91A potential coefficient model and a 2m rejection criteria are given in Table 3.

Table 3

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in the United States

	Vertical Datum	
	NGVD29	NAVD88
Mean Diff.	-26 cm	-72 cm
SD Diff.	±83 cm	±100 cm
No. Stations	321	321

The mean differences for NGVD29 and NAVD88 differ by 46 cm. Somewhat surprising is the somewhat poorer fit (100 cm vs 83 cm) when the NAVD88 datum is used. One would suspect that the NAVD88 heights should be more consistent with the ellipsoidal heights/gravimetric undulations than the NGVD29 heights which are given in a distorted datum.

Figure 4 of (ibid.) show the height differences between NAVD88 and NGVD29. This map shows that the heights with respect to the two datums are quite consistent in the eastern half of the country. Due to this, additional tests were made restricting the analysis to stations east of 254°E. The results of this analysis are given in Table 4. We see a substantial change (from Table 3) of the mean difference for stations on NAVD88 (-72 cm (Table 3) to -38 cm (Table 4)). The corresponding change in the NGVD29 difference is only 8 cm. The standard deviation of the undulation differences in going from NGVD29 to NAVD88 is smaller than the corresponding case in Table 3.

Table 4

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in the Eastern United States

	Vertical Datum	
	NGVD29	NAVD88
Mean Diff.	-18 cm	-38 cm
SD Diff.	±86 cm	±92 cm
No. Stations	180	180

These computations were repeated restricting the analysis to stations where the number of satellite passes was greater than 35 and the observation data was after June 15, 1978. The standard deviations would typically be reduced by about 10%. The mean differences would change by 7 to 12 cm.

The results of Table 3 make it unclear as to the appropriate mean difference to be associated with the two vertical datums. One can argue that a single parameter can not be used to represent the difference between the NGVD29 reference surface and the geoid because of the manner in which NGVD29 was adjusted. However, there is only a 8 cm difference between the total station set and the eastern region data. On the other hand, one would expect the greatest consistency with the NAVD88 system. Unfortunately the mean difference differs by 34 cm from the total station set in the eastern station set. With no other information available, one perhaps should use the results given in Table 3 understanding the value for NAVD88 is subject to change when more accurate heights in the NAVD88 system are used in the analysis. Specifically from Table 3 one estimates that the NGVD29 reference surface is located 26 cm below the geoid while the NAVD88 reference surface is 72 cm below the geoid.

Results in Australia

The heights available at the Doppler stations are approximate orthometric heights referred to the Australian Height Datum (AHD). The development of this datum is described by Leppert (1974) and Rizos et al. (1991). The levelling adjustment was carried out in 1971 using height constraints at 30 tide gauge stations around the coastline of Australia.

The Doppler stations were well distributed around Australia. The results of the analysis are given in Table 5 (AHD (M)). The mean difference of -68 cm suggests that the average AHD reference surface is located 68 cm below the geoid as defined by the JGM-2/OSU91A potential coefficient model. This magnitude conflicts with estimates of dynamic topography which suggests that mean sea level in the vicinity of Australia is above the geoid. (For example, see Figure 11 in Rapp, Wang, Pavlis (1991)).

Table 5

Mean and Standard Deviation of the Geometric and Gravimetric Undulation Difference for Doppler Stations in Australia and Tasmania

	AHD (M)	AHD (T)
Mean Diff.	-68 cm	-98 cm
SD Diff.	96	66
No. Stations	85	4

An attempt was made to determine the difference between the AHD as used in the mainland and the AHD used in Tasmania. This test was a follow-up to the study described by Rizos, Coleman and Ananga (1991). The first step was to use four Doppler stations in Tasmania that yielded a mean difference of -98 cm with

a standard deviation of ±66 cm (see Table 5 AHD(T)). This result implies that the AHD on the mainland has its reference surface 30 cm higher than the AHD in Tasmania. This value is similar to an updated (from Rizos, Coleman, Ananga (1991)) estimate of 40 cm (Coleman, 1993, private communication). A caveat here is that this analysis uses data distributed over all of Australia while the Rizos, Coleman, Ananga study considered the difference between the AHD (Victoria coast) and AHD (Tasmania).

Conclusions

This paper has demonstrated a procedure that estimates the separation between reference surfaces defined by several vertical datums and the geoid that is defined by geoid undulations computed from the JGM-2 (degree 2 to 70) and OSU91A (degree 71 to 360) potential coefficient model. The ellipsoidal heights of the stations are determined from Doppler positioning techniques after a transformation to the ITRF90 system. Although the accuracy of the vertical position derived from the Doppler positions is on the order of ±80 cm, the large number of stations on some vertical datums makes these tests meaningful.

In the analysis we analyzed data that can be associated with vertical datums in England, Germany, Australia, and the United States. We assume that the difference between the geoid and the reference surface of the vertical datum can be modeled by a single parameter. This would be appropriate if the datums were uniquely defined by a reference surface through a specified point. However, the adjustment of several of the datums was carried out by fixing heights at tide gauges distributed around the borders of the country. This fixing introduces a distortion in the reference surface (Laskowski, 1983) with respect to a single point related The reference surface. Our procedure then calculates a separation for some average reference surface representative of the area in which the stations are given.

The results for the differences in the origins of the vertical datums studied for this paper are shown in Figure 1. Not shown are the results for AHD (Tasmania) because of the few stations available. Two results for NAVD88 (U.S.) are shown because of the large differences in the result. One estimate is based on the complete station set after conversion from NGVD29 while the second estimate is based only on stations in the eastern part of the US. From Figure 1 one clearly sees the differences of the reference surfaces of a few vertical datums. It should be emphasized that the numerical value shown in tables and Figure 1 are dependent on many factors (station reference frame, geoid undulation model, equatorial radius, etc.).

The results (Table 3/4) for the two vertical datums in the U.S. are somewhat troubling. Why should the standard deviation of fit be poorer when the NAVD88 (a more rigorous datum) heights are used as opposed to the NGVD29 values? Why should the mean differences change so much (44 cm) when the results for the complete station set vs the eastern US station set of NAVD88 are considered? Additional study with more information on NAVD88 heights is needed.

Another area of concern is the magnitude of the geoid/datum separation for the Australian Height Datum. On the basis of oceanographic determinations of mean sea level one would expect the mean differences to be somewhat (~20 cm) positive instead of the -68 cm found in these results. On the encouraging side was the good agreement between the AHD(mainland)/AHD(Tasmania) difference between the 30 cm (AHD (M) higher)) and the recent estimate by Coleman of 40 cm.

To resolve some of these questions, more accurate computations are needed that would use more precise geocentric positions of stations on various vertical datums and more refined computation for geoid undulation values at the stations. Procedures to use this data for vertical datum definition and connections are described in Rapp and Balasubramania (1992).

And finally, the results of this study imply that height systems can be inconsistent by one or two meters. This is not new information as it is well known from studies of dynamic height variations in the oceans. There seems to be a clear need to develop a unique height system that can be tied to the ideal reference surface, the geoid. Given a sufficiently accurate representation of geoid undulations, orthometric heights can be derived from an ellipsoidal height computed from space positioning. Additional consideration is needed to define the type of orthometric height wanted, or to implement the determination of a normal height through a height anomaly calculation instead of a geoid undulation computation. Whatever the case, we are approaching, although not yet there, the situation of having sufficient gravity field information and ellipsoidal height values to define a world height system at the 50 - 100 cm accuracy level.

Acknowledgment

This study was supported by the Defense Mapping Agency and sponsored under Phillips Lab contract F19628-92-K-0026.

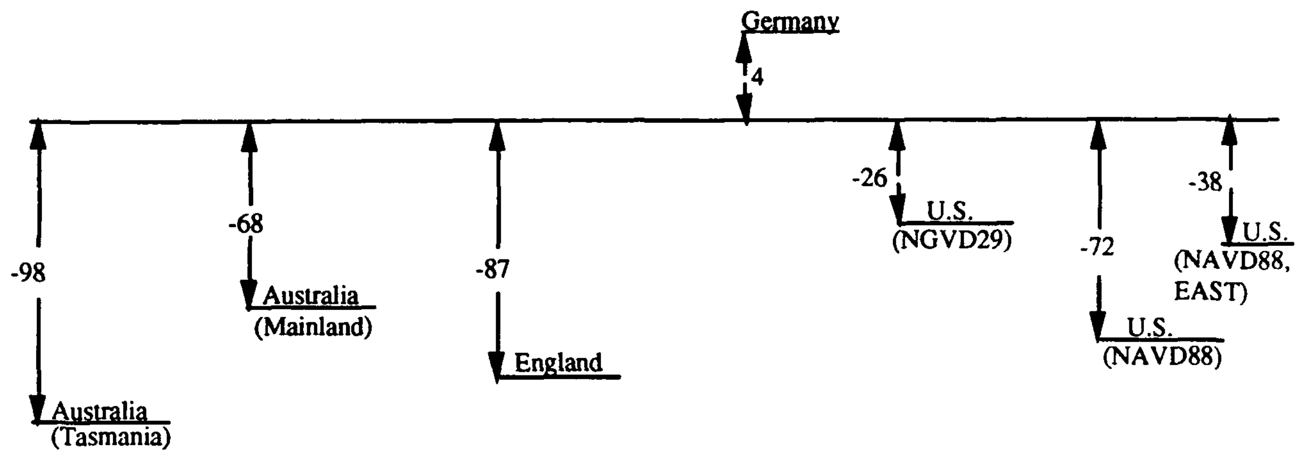

Fig. 1. Summary of Reference Surface Differences with Respect to the Geoid. Units are cm.

References

Ashkenazi, V. et al., The Determination of Mean Sea Level Using GPS, in Sea Surface Topography and the Geoid, proc. of IAG Symposium 104, Springer-Verlag, New York, Berlin, 1990.

Boucher, C. and Z. Altamimi, The IERS Terrestrial Reference System, in Proc. Sixth International Geodetic Symposium on Satellite Positioning, p. 47 - 55, DMA, 1992.

Heck, B. and R. Rummel, Strategies for Solving the Vertical Datum Problem Using Terrestrial and Satellite Geodetic Data, in Sea Surface Topography and the Geoid., Proc. of IAG Symposium 104, Springer-Verlag, New York, Berlin, 1990.

Laskowski, P., The Effect of Vertical Datum Inconsistencies on the Determination of Gravity Related Quantities, Report No. 349, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 1983.

Leppert, K., The Australian Levelling Adjustment, Progress to April 1970, Report on the Symposium on Coastal Geodesy, Munich, 20-24, July, p. 223, 1974.

Lerch, F.J. et al., Gravity Model Improvement for Topex/Poseidon, (abstract), EOS (American Geophysical Union), Vol. 74, No. 16, p. 96, 1993.

Pan, M. and L. Sjoberg, Baltic Sea Level Project with GPS, Bulletin Geodesique, 67, 51-59, 1993.

Rapp, R.H. and N.K. Pavlis, The Development and Analysis of Geopotential Coefficient Models to Spherical Harmonic Degree 360, J. Geophys., 95, B13, 21,885-21,911, 1990.

Rapp, R.H. , Y.M. Wang, N.K. Pavlis, The Ohio State 1991 Geopotential and Sea Surface Topography Harmonic Coefficient Models, Report No. 410, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 96 p., 1991.

Rapp, R.H. and N. Balasubramania, A Conceptual Formulation of a World Height System, Report No. 421, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 60 p., 1992.

Rapp, R.H., The Need and Prospects for a World Vertical Datum, presented at the Symposium, The Future of Terrestrial and Space Methods for Positioning, XVIII General Assembly, IUGG/IAG, Hamburg, 1983.

Rizos, C., R. Coleman, N. Ananga, The Bass Strait GPS Survey: Preliminary Results of an Experiment to Connect Australian Height Datums, Aust. J. Geod. Photogram. Surv., 55, 1-25, Dec. 1991.

Torge, W., Geodesy, de Gruyter, Berlin, New York, 1980.

Willis, P. et al., Connection of the two levelling system datum IGN69 and ODN through the Channel by using GPS and other techniques,

presented at the First International Workshop on Geodesy for the Europe-Africa fixed link feasibility studies in the Strait of Gibraltar, Madrid, 8-10 March 1989.

Weigel, G., Geoid Undulation Computations at Doppler Satellite Tracking Stations, manuscripta geodaetica, 18: 10-15, 1993.

Xu, P. and R. Rummel, A Quality Investigation of Global Vertical Datum Connection, Netherlands Geodetic Commission, New Series, No. 34, 1991.

Zilkoski, D., J. Richards, G. Young, Results of the General Adjustment of the North American Vertical Datum of 1988, Surveying and Land Information Systems, Vol. 52, No. 3, pp. 133-149, 1992.

3. Personnel

The principal investigator of this project is Richard H. Rapp, Professor Emeritus, Department of Geodetic Science and Surveying. Mr. Nagarajan Balasubramania, Graduate Research Associate, also participated in the research under this project. Mr. Balasubramania is a Ph.D. candidate at The Ohio State University.

4. Other Activities

During this project, Dr. M. Kumar, of the Defense Mapping Agency, visited us to learn about the research activities taking place. The first visit was on October 22, 1992 while the second visit was on March 23, 1993. For both visits, presentations were made to Dr. Kumar. Copies of the presentations were sent to Anestis Romaides at Phillips Laboratories (Code LW).

5. Travel

In August 1993, Dr. Rapp traveled, with support from this project, to the NASA/Goddard Space Flight Center for a meeting with NASA and DMA representations. This meeting concerned the possible joint activities of NASA, DMA, and Ohio State, in the production of a new gravity model that could be used for precise geoid determination. This travel was the only travel supported by the project. A follow-up meeting on the joint gravity field venture was held in October 1993 at NASA/Goddard. This meeting was also attended by Dr. Rapp. No travel funds from this project were used for this travel.

6. Presentations

Dr. Kumar and Dr. Rapp submitted an abstract for a presentation at the 1993 Fall Meeting of the American Geophysical Union. The abstract, as it appears in EOS, Trans. American Geophysical Union, Vol. 74, No. 43, October 26, 1993, is as follows:

Unification of World Height Systems

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There are several hundred vertical datums used worldwide and in some areas there is no information available that can be used to determine a height in the local system. Consequently, there is a substantial need to improve our vertical positioning capability for geodetic, mapping, and charting purposes. Numerous theories have been developed that enable connection and establishment of vertical datums. Such theories require data given in a specific datum with accurate space positioning techniques combined with dense gravity information. An alternative procedure proposed in this paper is to introduce the geoid as the unifying global vertical reference surface.

To locate the geoid with respect to a reference ellipsoid, geoid undulations must be accurately determined. Today a global geoid undulation accuracy estimate is about 26 cm in ocean areas. On land, the achievable accuracy is definitely poorer and in areas with scarce gravity data the accuracy may be even 2-5 m. To achieve an accurate determination of the global geoid for vertical datum purposes improvement in our knowledge of the Earth's gravity field is needed.

Thus, as a first step, efforts are being made to develop a new degree 360 potential coefficient model combining all the available satellite and terrestrial gravity data. This new model should then enable the determination of geoid undulations to an accuracy of 50 to 100 cm in land areas

with substantially better accuracy over ocean areas. The new geoid is then intended to be implemented as the vertical reference surface for the World Geodetic System (WGS) 1984. These geoidal undulations would also enable the determination of orthometric heights from space positioning systems, e.g., GPS, with respect to a consistent zero reference surface globally. In effect, the improved geoid will result in the realization of WGS84 as a complete three-dimensional reference system. The connection of this global height system to local vertical datums can also be then easily achieved. The technical approach to develop such a united world height system will be described in this presentation.

7. Future Activities

Most of the research goals defined in the original proposal have been accomplished during this contract. This report has described two aspects of the research carried out. Additional details of the research by Balasubramania will be given in his dissertation which should be completed by the 2nd quarter of 1994.

The results of this research clearly shows the need to more clearly define our vertical datums. The numerical results demonstrate the origin inconsistencies that exist in a number of current datums. Although these inconsistencies are at the ±1 m level, they are of increasing concern in today's high precision geodetic positioning. We have seen that one of the difficulties in connecting our datums in the lack of a sufficiently accurate gravity model for the Earth. Current models can be used to derive geoid undulations that have an accuracy of ±0.5 to ±5 m. With improved data both satellite and surface gravity data, an improvement in our representation of the Earth's gravitational potential can be anticipated. This improvement could lead to the ideal reference surface, the geoid, to be used for a global vertical reference system. Ultimately, this surface could be the reference for a World Height System.

References

Boucher, C. and Z. Altamimi, ITRF 90 and Other Realizations of the Terrestrial Reference System for 1990, IERS Technical Note 9, Observatoire de Paris, Dec. 1991.

Despotakis, V.K., Geoid Undulation Computation at Laser Tracking Stations, Report No. 383, Geodetic Science and Surveying, The Ohio State University, Columbus, 1983.

Heiskanen, W. and H. Moritz, Physical Geodesy, W.H. Freeman and Co., San Francisco, 1967.

Milbert, D.G., An Accuracy Assessment of the GEOID90 Geoid Height Model for the Commonwealth of Virginia, NOAA, June 1991.

Milbert, D.G., GEOID 90: A High-Resolution Geoid for the United States, EOS, Trans. American Geophysical Union, Vol. 72, No. 49, 1991b.

Morgan, P., An Analysis of the Australian Height Datum: 1971, The Australian Surveyor, March 1992, Vol. 37, No. 1.

Moritz, Helmert, Advanced Physical Geodesy, Abacus Press, Townbridge Wells Kent, 1980.

Pavlis, N.K., Modelling and Estimation of a Low Degree Geopotential Model from Terrestrial Gravity Data, Report No. 386, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 1988.

Rapp, R.H. and Balasubramania, N., A Conceptual Formulation of a World Height System, Report No. 421, Dept. of Geodetic Science and Surveying, The Ohio State University, Columbus, 1992.

Rapp, R.H., Computation and Accuracy of Global Geoid Undulation Models, presented at the Sixth International Geodetic Symposium on Satellite Positioning, Columbus, Ohio, 17-20 March 1992.

Rapp, R.H. and Pavlis, N.K., The Development and Analysis of Geopotential Coefficient Models to Spherical Harmonic Degree 360, Journal of Geophysical Research 95 (B13), pp. 21885-21911, 1990.

Rapp, R.H., Separation Between Reference Surfaces of Selected Vertical Datums, November 1993, to appear in Bulletin Geodesique/manuscripta geodaetica (also see Section 2.2).

Sideris, M.G. and Rene Forsberg, Review of Geoid Prediction Methods in Mountainous Regions, Determination of the Geoid - Present and Future, IAG Symposia 106, pp. 51-62, 1991.

Torge, W., G. Weber, and H.G. Wenzel, 6'x10' Free Air Gravity Anomalies of Europe Including Marine Areas, Presented to the XVIII IUGG General Assembly, Hamburg, 15-27 August 1983.

Wang, Y.M., Comment on Proper Use of the Terrain Correction of Height Anomalies, The Ohio State University, Columbus, 1993.